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MODIFICATION OF RATIO ESTIMATOR UNDER TWO PHASE SAMPLING

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Abstract: In this paper, modification of Adebola and Adegoke's report on ratio estimator was suggested. The modified estimator was obtained through transformation in two cases using sample mean of auxiliary variables. Case one was when the second sample was drawn from the first sample why case two was when the second sample was drawn from the population. The bias and mean square error (MSE) of the modified ratio estimator in the two cases were obtained. The theoretical and numerical validity of the modified ratio estimator under the two cases were determined to show its superiority over some considered existing related ratio estimators. Numerical results shows that the modified ratio estimator under the two cases were more efficient than the considered existing related estimators.

Keywords: Ratio estimator, two-phase sampling, bias, mean square error

Introduction

Sampling is a method or technique of drawing sample from the population. It is used whenever the population is large and the complete enumeration is very time consuming and costly. Parameters of the population are estimated through their appropriate estimators using the information supplied by the sample and their large sample properties are studied up to a certain order of approximation (Cochran, 1977).

In sample surveys, auxiliary information is used at both selections as well as estimation stages to improve the efficiency of the estimators. The use of auxiliary information has become indispensable for improving the precision of the estimators of population parameters such as the mean and variance of a variable under study. A great variety of techniques such as the ratio, product and regression methods of estimation are commonly known in this regard. Auxiliary information can be used either at the design stage or at the estimation stage or at both the stages (Cochran, 1940; Okafor, 2002). Use of auxiliary information has been in practice to increase the efficiency of the estimators. When the population means of an auxiliary variate is known, so many estimators for population parameter(s) of study variate have been discussed in the literature. When correlation between study variate and auxiliary variate is positive (high) ratio method of estimation is used (Cochran, 1940). On the other hand, if the correlation is negative, product method of estimation is preferred (Robson, 1957; Murthy, 1967). In practice, information on coefficient of variation (CV) of an auxiliary variate is seldom known. Sisodia and Dwivedi (1981) suggested a modified ratio estimator for population mean of the study variate. Later on, Upadhyaya and Singh (1999) derived another ratio type estimator using coefficient of variation and coefficient of kurtosis of the auxiliary variate. Singh (1967) utilized information on two auxiliary variate x_1 and x_2 and suggested a ratio-cum-product estimator for population mean. Singh and Tailor (2005) utilized known correlation coefficient between auxiliary variates $(p_{x_1 x_2})$ x_1 and x_2.

Singh and Tailor (2005) motivates authors to suggest ratio-cum-product estimators of population mean utilizing the information on co-efficient of variation of auxiliary variates i.e. C_{x_1} and C_{x_2} and co-efficient of kurtosis of auxiliary variates $\beta_2(x_1)$ and $\beta_2(x_2)$ besides the population means ($\bar{X}_1$ and $\bar{X}_2$) of auxiliary variates x_1 and x_2. Murthy 1964 suggested the use of ratio estimator $\bar{y}_p$ when $\frac{pc_y}{c_x} > \frac{1}{2}$ and unbiased estimator $\bar{y}$ when $-\frac{1}{2} \le p\frac{c_y}{c_x} \le \frac{1}{2}$, where c_y, c_x and p are coefficients of variation of y, x and correlation between y and x respectively.

Suppose that simple random sample without replacement SRSWOR of n units is drawn from a population of N units to estimate the population mean $\bar{Y} = \frac{1}{N}\sum_{i=1}^{N} y_i$ of the study variable Y. All the sample units are observed for the variables Y and X. Let (y_i, x_i) where i =1,2,3,…,n denote the set of the observation for the study variable Y and X. Let the sample means ($\bar{x}, \bar{y}$) be unbiased of the population means of the auxiliary variable $\bar{X}$ and study variable $\bar{Y}$ based on the n observations. The use of auxiliary information in sample surveys was extensively discussed in well-known classical text books such as Cochran (1977), Sukhatme and Sukhatme (1970), Sukhatme *et al.* (1984), Murthy (1967) and Yates (1960) among others. Cochran (1940) was the first to show the contribution of known auxiliary information in improving the efficiency of the estimator of population mean $\bar{Y}$ in survey sampling. Assuming the population mean $\bar{X}$ of the auxiliary variable is known, he introduced a ratio estimator of population mean $\bar{Y}$ defined as $\bar{y}_R = \bar{y}(\frac{\bar{X}}{\bar{x}})$.

In the ratio estimator, an auxiliary variate x_i , correlated with y_i, is obtained for each unit in the sample. The population total X of the x_i must be known. In practice, x_i is often the value of y_i at some previous time when a complete census was taken. The aim in this method is to obtain increased precision by taking advantage of the correlation between y_i and x_i. At present, we assume simple random sampling. In survey sampling, the utilization of auxiliary information is frequently acknowledged to higher the accuracy of the estimation of population characteristics. This has motivated some researchers to look for different techniques to form ratio type estimators whose mean square errors approximate to that of the approximate mean square error of the linear regression estimate in large samples. Several well known procedures use auxiliary information at the estimate stage. This is most commonly used way of utilizing auxiliary information which gives rise to some estimators that are known today, and in under certain conditions, these estimators are more efficient than the estimator's based on simple random sampling The ratio estimator of the form, $\bar{y}_R$ is given as ;

$$\bar{y}_R = \bar{y}\frac{\bar{X}}{\bar{x}}; \bar{X} \neq o \qquad (1)$$

where $\bar{y}$ and $\bar{x}$ are sample means of the characteristics under study and auxiliary characteristics respectively based on sample drawn under simple random sample design and $\bar{X}$ is the population mean of the auxiliary characteristics X. The bias and MSE are given as,

$$B(\bar{Y}_R) = \bar{Y}(\frac{1}{n} - \frac{1}{N})(S_x^2 \rho_{yx} S_y S_x) \qquad (2)$$

$$M(\bar{Y}_R) = \bar{Y}^2(\frac{1}{n} - \frac{1}{N})(S_y^2 + \hat{R}^2 S_x^2 - 2\hat{R}\rho_{yx} S_y S_x) \qquad (3)$$

Kumar (2006) suggested ratio estimator in double sampling as;

$$\bar{Y}_R^{*d} = \bar{y}\frac{\bar{x}^{*d}}{\bar{x}_1} \qquad (4)$$

and

The Bias and Mean Squared Error as;

$$B(\bar{Y}_R^{*d})_1 = -\bar{Y}\frac{1-f}{n} g k_{yx} C_x^2 \qquad (5)$$

$$M(\bar{Y}_R^{*d})_1 = \bar{Y}^2\{\frac{1-f}{n}C_y^2 + \frac{1-f}{n}C_x^2 g(g - 2k_{yx})\} \qquad (6)$$

Srivenkataramana (1980) obtained dual to ratio estimator as;

$$\bar{Y}_R^{(d)} = \bar{y}\left(\frac{\bar{x}^*}{\bar{X}}\right) \qquad (`7)$$

where $\bar{x}^* = (N\bar{X} - n\bar{x})/(N-n)$

The bias and mean squared error are respectively given as

$$B(\bar{y}_R^{(d)}) = \frac{1-f}{n}\bar{Y} g C_x^2 (1-k) \qquad (8)$$

$$MSE(\bar{y}_R^{(d)}) = \frac{1-f}{n}\bar{Y}^2[C_y^2 + gC_x^2(g-2k)] \qquad (9)$$

Adebola and Adegoke (2015), in their work suggested estimator under sampling scheme as;

$$\bar{Y}_{pd}^* = \bar{y}\frac{\bar{X}}{\bar{x}^*} + \alpha(\bar{X} - \bar{x}^*) \qquad (10)$$

The bias and mean squared error are respectively given as;

$$Bias(\bar{Y}_{pd}^*) = \frac{1-f}{n}(g^2\bar{Y}^2\frac{S_X^2}{\bar{X}^2} + \bar{Y}g\frac{S_{XY}}{\bar{X}\bar{Y}}) \qquad (11)$$

$$MSE(\bar{Y}_{pd}^*) = (\frac{1-f}{n})S_Y^2(1-\rho^2) \qquad (12)$$

Theoretical and empirical studies were carried out and the conditions for the efficiency of their estimator over some existing estimators were established and the numerical results revealed that their estimator performed better.

The modified estimator

Having studied the ratio estimator suggested by Adebola and Adegoke (2015) as:

$$\bar{y}_{pd}^* = \bar{y}\frac{\bar{X}}{\bar{x}^*} + \alpha(\bar{X} - \bar{x}^*) \qquad (13)$$

Thus, the general modified estimator under two-phase sampling is given as:

$$\bar{y}_{pdIA1}^* = \bar{y}_2\frac{\bar{x}_1}{\bar{x}_2^*} + \alpha_1(\bar{x}_1 - \bar{x}_2^*) \qquad (14)$$

where

$$\bar{x}_2^* = \frac{n_1\bar{x}_1 - n_2\bar{x}_2}{n_1 - n_2} \qquad (15)$$

$\bar{y} =$ Sample mean of the study variable

$\bar{x}_1 =$ Sample mean of the auxiliary variable

$\bar{x}_2^* =$ Sample mean based on sample yet to drawn

$\alpha_1 =$ Unknown weight $(0 < \alpha_1 < 1)$

The above general modified estimator is based on the following assumptions;

(i) $\bar{x}_2^* \neq 0$

(ii) $\rho_{xy} > 0$

(iii) 0< α_1 <1

(iv) $n_2 < \frac{1}{2}n_1$

Bias, MSE and optimum of $\bar{y}_{pdIA1}^*$

$$\bar{y}_{pdIA1}^* = \bar{y}_2\frac{\bar{x}_1}{\bar{x}_2^*} + \alpha_1(\bar{x}_1 - \bar{x}_2^*)$$

We write $e_0 = \frac{\bar{y}_2 - \bar{Y}}{\bar{Y}}$ $\qquad e_1 = \frac{\bar{x}_1 - \bar{X}}{\bar{X}}$ $\qquad e_2 = \frac{\bar{x}_2 - \bar{X}}{\bar{X}}$

$\bar{y}_2 = (1+e_0)\bar{Y}$ $\qquad \bar{x}_1 = (1+e_1)\bar{X}$ $\qquad \bar{x}_2 = (1+e_2)\bar{X}$

By substituting $\bar{x}_1 = (1+e_1)\bar{X}$ and $\bar{x}_2 = (1+e_2)\bar{X}$ in to equation (15), we have;

$$\bar{x}_2^* = \frac{n_1(1+e_1)\bar{X} - n_2(1+e_2)\bar{X}}{n_1 - n_2}$$

$$= \frac{\bar{X}(n_1 + n_1 e_1 - n_2 - n_2 e_2)}{n_1 - n_2}$$

$$= \bar{X}[\frac{(n_1 - n_2) + n_1 e_1 - n_2 e_2)}{n_1 - n_2}] \qquad = \bar{X}[1 + \frac{n_1}{n_1 - n_2}e_1 - \frac{n_2}{n_1 - n_2}e_2]$$

$$= \bar{X}[1 + h_1 e_1 - h_2 e_2]$$

where

$$h_1 = \frac{n_1}{n_1 - n_2} \qquad h_2 = \frac{n_2}{n_1 - n_2}$$

$$\bar{y}_{pdIA1} = (1+e_0)\bar{Y}\frac{(1+e_1)\bar{X}}{\bar{X}[1 + h_1 e_1 - h_2 e_2]} + \alpha_1[(1+e_1)\bar{X} - \bar{X}(1 + h_1 e_1 - h_2 e_2)]$$

$$= \bar{Y}(1+e_0)(1+e_1)(1 + h_1 e_1 - h_2 e_2)^{-1} + \bar{X}\alpha_1[e_1 - h_1 e_1 + h_2 e_2]$$

$$= \bar{y}[1+e_0][1+e_1][1 - (h_1 e_1 - h_2 e_2) + (h_1 e_1 - h_2 e_2)^2 ....] + \bar{X}\alpha_1[(1-h_1)e_1 + h_2 e_2]$$

$$= \bar{Y}[1+e_0][1+e_1][1 - h_1 e_1 + h_2 e_2 + h_1^2 e_1^2 + h_2^2 e_2^2 - 2h_1 h_2 e_1 e_2] + \bar{X}\alpha_1((1-h_1)e_1 + h_2 e_2)$$

$$= \bar{Y}(1+e_0)[1 - h_1 e_1 + h_2 e_2 + h_1^2 e_1^2 + h_2^2 e_2^2 - 2h_1 h_2 e_1 e_2 + e_1 - h_1 e_1^2 + h_1 e_1 e_2] + \bar{X}\alpha_1((1-h_1)e_1 + h_2 e_2)$$

$$= \bar{Y}[1+e_0][1 + (1-h_1)e_1 + (h_1^2 - h_1)e_1^2 + (h_2 - 2h_1 h_2)e_1 e_2 + h_2 e_2 + h_2^2 e_2^2) + \bar{X}\alpha_1[(1-h_1)e_1 + h_2 e_2]$$

$$= \bar{Y}[1 + (1-h_1)e_1 + (h_1^2 - h_1)e_1^2 + (h_2 - 2h_1 h_2)e_1 e_2 + h_2 e_2 + h_2^2 e_2^2 + e_0 + (1-h_1)e_0 e_1] + \bar{X}\alpha_1[(1-h_1)e_1 + h_2 e_2]$$

$$\bar{Y}_{pdIA1}^* - \bar{Y} = \bar{Y}[(1-h_1)e_1 + (h_1^2 - h_1)e_1^2 + (h_2 - 2h_1 h_2)e_1 e_2 + h_2 e_2 + h_2^2 e_2^2 + e_0 + (1-h_1)e_0 e_1] + \bar{X}\alpha_1[(1-h_1)e_1 + h_2 e_2]$$

To get the bias, take expectation;

$$E[\bar{Y}_{pdIA1}^* - \bar{Y}] = Bias(\bar{Y}_{pdIA1}^*) \tag{16}$$

Under case 1: Where $S_2 \subset S_1$ (Sample 2 drawn from Sample 1)

$$E(e_0) = E(e_1) = E(e_2) = 0$$

$$E(e_0^2) = (\tfrac{1}{n_2} - \tfrac{1}{n_1})\tfrac{S_y^2}{\bar{Y}^2} = f_0 \tfrac{S_y^2}{\bar{Y}^2} = f_0 C_y^2$$

$$E(e_1^2) = (\frac{1}{n} - \frac{1}{N})\frac{S_x^2}{\bar{X}^2} = f_1 \frac{S_x^2}{\bar{X}^2} = f_1 C_x^2$$

$$E(e_2^2) = (\tfrac{1}{n_2} - \tfrac{1}{n_1})\tfrac{S_x^2}{\bar{X}^2} = f_0 \tfrac{S_x^2}{\bar{X}^2} = f_0 C_x^2$$

$$E(e_0 e_1) = f_1 \ell_{xy} \tfrac{S_x S_y}{\bar{X}\bar{Y}} = f_1 \ell_{xy} C_y C_x$$

$$E(e_0 e_2) = f_0 \ell xy \tfrac{S_x S_y}{\bar{X}\bar{Y}} = f_0 \ell_{xy} C_y C_x$$

$$E(e_1 e_2) = f_1 \tfrac{S_x^2}{\bar{X}^2} = f_1 C_x^2$$

Under case 2: $S_2 \subset \Omega$ (Sample 2 drawn from the population)

$$E(e_0) = E(e_1) = E(e_2) = 0$$

$$E(e_0^2) = f_0 \tfrac{S_y^2}{\bar{Y}^2} = f_0 C_y^2$$

$$E(e_1^2) = f_1 \tfrac{S_x^2}{\bar{X}^2} = f_1 C_x^2$$

$$E(e_2^2) = f_0 \tfrac{S_x^2}{\bar{X}^2} = f_0 C_x^2$$

$$E(e_0 e_1) = 0$$

$$E(e_0 e_2) = f_1 \ell_{xy} \tfrac{S_x S_y}{\bar{X}\bar{Y}} = f_1 \rho_{xy} C_x C_y$$

$$E(e_1 e_2) = 0$$

Under case 1: ($S_2 \subset S_1$)

The bias $\bar{Y}_{pdIA1}^*$ is given as

$$Bias(\bar{Y}_{pdIA1}^*) = \bar{Y}[(h_1^2 - h_1) f_0 C_x^2 + (h_2 - 2h_1 h_2) f_1 C_x^2 + h_2^2 f_0 C_x^2 + (1-h_1) f_1 \rho_{xy} C_x C_y] \tag{17}$$

The MSE of $\bar{Y}_{pdIA1}^*$ is given as

$$MSE(\bar{Y}_{pdIA1}^*) = E\{[(1-h_1)^2 e_1^2 + h_2^2 e_2^2 + e_0^2]\bar{Y}^2 + \bar{X}^2\alpha_1^2[(1-h_1)^2 e_1^2 + h_2^2 e_2^2] + 2\bar{Y}^2[(1-h_1)h_2 e_1 e_2 + (1-h_1)e_1 e_0 + h_2 e_0 e_2] +$$

$$\bar{X}^2\alpha_1^2[2(1-h_1)h_2e_1e_2]+2\bar{Y}\bar{X}\alpha_1[(1-h_1)^2e_1^2+(1-h_1)h_2e_1e_2+h_2(1-h_1)e_1e_2+h_2^2e_2^2+(1-h_1)e_0e_1+h_2e_0e_2]\}$$

$$=[(1-h_1)^2 f_1C_x^2+h_2^2 f_0C_x^2+f_0C_y^2]\bar{Y}^2+\bar{X}^2\alpha_1^2[(1-h_1)^2 f_1C_x^2+h_2^2 f_0C_x^2]+$$
$$2\bar{Y}^2[(1-h_1)h_2 f_1C_x^2+(1-h_1)f_1\rho_{xy}C_xC_y+h_2f_0\rho_{xy}C_xC_y]$$
$$+2\bar{X}^2\alpha_1^2(1-h_1)h_2 f_1C_x^2+2\bar{Y}\bar{X}\alpha_1[(1-h_1)^2 f_1C_x^2+2(1-h_1)h_2 f_1C_x^2+h_2^2 f_0C_x^2+(1-h_1)f_1\rho_{xy}C_xC_y+h_2f_0\rho_{xy}C_xC_y]$$

$$=\bar{Y}^2[f_0C_y^2+((1-h_1)^2 f_1+h_2^2 f_0+2(1-h_1)h_2 f_1)C_x^2+2((1-h_1)f_1+h_2f_0)\rho_{xy}C_xC_y]+$$
$$\bar{X}^2\alpha_1^2[(1-h_1)^2 f_1+h_2^2 f_0+2(1-h_1)h_2 f_1]C_x^2+2\bar{Y}\bar{X}\alpha_1[[(1-h_1)^2 f_1+2(1-h_1)h_2 f_1+h_2^2 f_0]C_x^2+$$
$$[(1-h_1)f_1+h_2f_0]\rho_{xy}C_xC_y]$$

The optimum value of the MSE ($\bar{Y}^*_{pdIA1}$) is obtained as follows,

$$\frac{\partial MSE(\bar{Y}^*_{pdIA1})}{\partial\alpha_1}=2\alpha_1\bar{X}^2[(1-h_1)^2 f_1+h_2^2 f_0+2(1-h_1)h_2 f_1]C_x^2+2\bar{Y}\bar{X}[[(1-h_1)^2 f_1+2(1-h_1)h_2 f_1+h_2^2 f_0]C_x^2+$$
$$[(1-h_1)f_1+h_2f_0]\rho_{xy}C_xC_y]=0$$
$$2\alpha_1\bar{X}^2[(1-h_1)^2 f_1+h_2^2 f_0+2(1-h_1)h_2 f_1]C_x^2=$$
$$-2\bar{Y}\bar{X}[[(1-h_1)^2 f_1+2(1-h_1)h_2 f_1+h_2^2 f_0]C_x^2+[(1-h_1)f_1+h_2f_0]\rho_{xy}C_xC_y]$$

$$\alpha_1=\frac{-2\bar{Y}[[(1-h_1)^2 f_1+2(1-h_1)h_2 f_1+h_2^2 f_0]C_x+[(1-h_1)f_1+h_2f_0]\rho_{xy}C_y]}{\bar{X}[(1-h_1)^2 f_1+h_2^2 f_0+2(1-h_1)h_2 f_1]C_x} \qquad (18)$$

$$=\bar{Y}^2[f_0C_y^2+..........+\frac{4\bar{Y}^2[[(1-h_1)^2 f_1+2(1-h_1)h_2 f_1+h_2f_0]C_x+[(1-h_1)f_1+h_2f_0]\rho_{xy}C_y]^2}{[(1-h_1)^2 f_1+h_2^2 f_0+2(1-h_1)h_2 f_1]}$$
$$-\frac{4\bar{Y}^2[[(1-h_1)^2 f_1+2(1-h_1)h_2 f_1+h_2^2 f_0]C_x+[(1-h_1)f_1+h_2f_0]\rho_{xy}C_y]^2}{[(1-h_1)^2 f_1+h_2^2 f_0+2(1-h_1)h_2 f_1]}$$

$$MSE(\bar{Y}^*_{pdIA1})^{opt}=\bar{Y}^2[f_0C_y^2+((1-h_1)^2 f_1+h_2 f_0+2(1-h_1)h_2 f_1)C_x^2+2((1-h_1)f_1+h_2f_0)\rho_{xy}C_xC_y] \qquad (19)$$

Under Case 2

$$Y^*_{pdIA1}=\bar{Y}[(1-h_1)e_1+(h_2^2-h_1)e_1^2+(h_2-2h_1h_2)e_1e_2+h_2e_2+h_2^2e_2^2+e_0+(1-h_1)e_0e_1]+\bar{X}\alpha_1[(1-h_1)e_1+h_2e_2]$$

Take expectation to fit bias under case 2

$$Bias(\bar{Y}^*_{pdIA1})_{II}=\bar{Y}[f_1(h_1^2-h_1)C_x^2+h_2^2 f_0C_x^2] \qquad (20)$$

Square $(\bar{Y}^*_{pdIA1})_{II}$ and take expectation, the MSE $(\bar{Y}^*_{pdIA1})_{II}$ under case 2 is obtained as:

$$MSE(\bar{Y}^*_{pdIA1})_{II}=\bar{Y}^2[(1-h_1)^2 f_1C_x^2+h_2^2 f_0C_x^2+f_0C_y^2+h_2f_1\ell_{xy}C_yC_x]+\bar{X}^2\alpha_1^2[(1-h_1)^2 f_1C_x^2+h_2^2 f_0C_x^2]+$$
$$2\bar{Y}\bar{X}\alpha_1[(1-h_1)^2 f_1C_x^2+h_2^2 f_0C_x^2+h_2f_1\ell_{xy}C_yC_x]$$

$$\frac{\partial MSE(\bar{Y}^*_{pdIA1})_{II}}{\partial\alpha_1}=\bar{X}2\alpha_1[(1-h_1)^2 f_1C_x^2+h_2^2 f_0C_x^2]+2\bar{Y}\bar{X}[(1-h_1)^2 f_1C_x^2+h_2^2 f_0C_x^2+h_2f_1\ell_{xy}C_yC_x]=0$$

$$\alpha_1=\frac{-\bar{Y}[(1-h_1)^2 f_1C_x^2+h_2^2 f_0C_x^2+h_2f_1\ell_{xy}C_yC_x]}{\bar{X}[(1-h_1)^2 f_1C_x^2+h_2^2 f_0C_x^2]} \qquad (21)$$

$$MSE(\bar{Y}^*_{pdIA1})^{Opt}_{II}=\bar{Y}^2\frac{[[f_0C_y^2+(1-h_1)^2 f_1C_x^2+h_2^2 f_0C_x^2+2h_2f_1\ell_{xy}C_yC_x]-[(1-h_1)^2 f_1C_x^2+h_2^2 f_0C_x^2+h_2f_1\ell_{xy}C_yC_x]^2]}{[(1-h_1)^2 f_1C_x^2+h_2^2 f_0C_x^2]} \qquad (22)$$

Empirical study

To analyze the performance of the modified ratio estimator under the two cases in comparison to other existing related estimators, five natural data sets were considered. The sources of the data, the nature of the variates y and x and the values of the various parameters were given as follows:

Data 1: Cochran (1977)

$$\bar{X} = 58.8, \bar{Y} = 101.1, C_X = 0.1281, C_Y = 0.1445, \rho = 0.65, N = 20, n_2 = 5, n_1 = 12$$

Data 2: Das (1988)

$$\bar{X} = 25.11, \bar{Y} = 39.07, C_X = 1.6198, C_Y = 1.4451, \rho = 0.72, N = 278, n_2 = 80, n_1 = 180.$$

Data 3: Nachtsheim *et al.* (2004)

$$\bar{X} = 906.76, \bar{Y} = 119.50, C_X = 1.7501, C_Y = 1.9955, \rho = 0.956, N = 440, n_2 = 37, n_1 = 88$$

Data 4: Nachtsheim *et al.* (2004)

$$\bar{X} = 76.95, \bar{Y} = 2.9773, C_X = 0.242157, C_Y = 0.213123, \rho = 0.398, N = 705, n_2 = 28, n_1 = 141$$

Data 5: Singh and Audu (2015)

$$\bar{X} = 11.90, \bar{Y} = 5.60, C_X = 0.0721, C_Y = 0.1034, \rho = 0.09, N = 82, n_2 = 20, n_1 = 43$$

Efficiency comparisons

Table 1 shows the biases, mean square errors and percentage relative efficiency of the modified and some related ratio estimators using Data 1. The results revealed that all the considered estimators are biased with exception of sample mean. The results also revealed that the modified estimators have minimum MSE and highest PRE among the considered estimators. This implies that the modified estimators are more efficient.

Table 1: Bias, MSE and PRE of $\bar{y}^{*}_{pdIA1}$ and some related ratio estimators using Data 1:

Estimators	Bias	MSE	PRE
Sample mean	0	32.01321	100
Cochran (1942)	0.0663896	20.27828	157.8695
Srivenkataramana (1980)	0.02212987	22.5107	142.2133
Kumar *et al*. (2006)	-0.1013679	18.49665	173.0758
Sharma & Tailor (2010)	0.01548313	24.62237	130.0168
Adebola & Adegoke (2015)	2.856255	18.48763	173.1602
Modified $\bar{y}^{*}_{pdIA1}$ (case 1)	-0.2262593	16.19052	197.7282
Modified $\bar{y}^{*}_{pdIA1}$ (case 2)	0.3357523	17.29103	185.1435

Table 2: Bias, MSE and PRE of $\bar{y}^{*}_{pdIA1}$ and some related ratio estimators using Data 2

Estimators	Bias	MSE	PRE
Sample mean	0	28.38004	100
Cochran (1942)	0.3264072	18.22889	155.6871
Srivenkataramana (1980)	0.1318817	15.69275	180.8481
Kumar *et al*. (2006)	-0.365816	14.55407	194.9973
Sharma & Tailor (2010)	-0.01162667	18.48367	153.5412
Adebola & Adegoke (2015)	6.057752	13.66783	207.6412
Modified $\bar{y}^{*}_{pdIA1}$ (case I)	-0.8865601	9.055789	313.3912
Modified $\bar{y}^{*}_{pdIA1}$ (case 2)	1.4807	13.87189	204.5868

Table 2 shows the biases, mean square errors and percentage relative efficiency of the modified and some related ratio estimators using Data 2. The results revealed that all the considered estimators are biased except the sample mean. The results also revealed that the modified estimator under case 1 has the minimum MSE and highest PRE among the considered estimators.

Table 3 shows the biases, mean square errors and percentage relative efficiency of the modified and some related ratio estimators using Data 3. The results revealed that all the considered estimators are biased except the sample mean. The results also revealed that the modified estimator has the minimum MSE and highest PRE among the considered estimators. This implies that the modified estimators are more efficient. Table 4 shows the biases, mean square errors and percentage relative efficiency of the modified and some related ratio estimators using Data 4. The results revealed that all the considered estimators are biased except the sample mean. The results also revealed that the modified estimators have the minimum MSE and highest PRE among the considered estimators. This implies that the modified estimators are more efficient.

Table 3: Bias, MSE and PRE of $\bar{y}^{*}_{pdIA1}$ and some related ratio estimators using Data 3:

Estimators	Bias	MSE	PRE
Sample mean	0	1407.635	100
Cochran (1942)	-0.815891	129.9265	1083.408
Srivenkataramana (1980)	-0.0749081	1200.048	117.2982
Kumar *et al*. (2006)	-4.533752	265.0437	531.0953
Sharma & Tailor (2010)	-0.8068819	1276.283	110.2917
Adebola & Adegoke (2015)	10.03328	121.1467	1161.926
Modified $\bar{y}^{*}_{pdIA1}$ (case I)	-13.08171	52.51127	2680.633
Modified $\bar{y}^{*}_{pdIA1}$ (case 2)	10.19414	102.7619	1369.802

Table 4: Bias, MSE and PRE of $\bar{y}^{*}_{pdIA1}$ and some related ratio estimators using Data 4:

Estimators	Bias	MSE	PRE
Sample mean	0	0.01380853	100
Cochran (1942)	0.003890307	0.01914665	72.11982
Srivenkataramana (1980)	0.000160899	0.01332249	103.6482
Kumar *et al*. (2006)	-0.0004337245	0.01180847	116.9375
Sharma & Tailor (2010)	0.001194859	0.0134029	103.0264
Adebola & Adegoke (2015)	0.0001172392	0.0116212	118.8219
Modified $\bar{y}^{*}_{pdIA1}$ (case 1)	-0.0008867941	0.01083106	127.49
Modified $\bar{y}^{*}_{pdIA1}$ (case 2)	0.00185185	0.01033182	133.6505

Table 5: Bias, MSE and PRE of $\bar{y}^{*}_{pdIA1}$ and some related ratio estimators using Data 5

Estimators	Bias	MSE	PRE
Sample mean	0	0.0126755	100
Cochran (1942)	0.0009584937	0.01724759	73.49139
Srivenkataramana (1980)	0.0003091915	0.01280361	98.99942
Kumar *et al*. (2006)	-8.738118e-05	0.01595221	79.45917
Sharma & Tailor (2010)	-0.0003327121	0.01276612	99.29009
Adebola & Adegoke (2015)	0.0006871363	0.01257282	100.8166

Modified $\overline{y}^{*}_{pdIA1}$ (case 1)	-0.0008904497	0.007198804	176.0778
Modified $\overline{y}^{*}_{pdIA1}$ (case 2)	0.001854397	0.008906854	142.3117

Table 5 shows the biases, mean square errors and percentage relative efficiency of the modified and some related ratio estimators using Data 5. The results revealed that all the considered estimators are biased except the sample mean. The results also revealed that the modified estimator has the minimum MSE and highest PRE among the considered estimators. This implies that the modified estimators are more efficient.

Conclusion

In this paper, we modified a ratio estimator for the estimation of the sample mean using two phase sampling scheme under two cases. The numerical comparison of the estimators among considered existing related estimators in Tables 1 to 5 shows that the performances of the modified ratio estimator under the two cases perform better and more efficient than the existing considered related estimators. Hence, it is recommended for usage in sample survey.

Conflict of Interest

Authors declare that there is no conflict of interest reported on this work.

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